BANSAL CLASSES
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FACULTY SELECTION TEST

MATHEMATICS

Time: 90 Min. **Max. Marks: 200**

GENERAL INSTRUCTIONS

1. Write your Name in the Space Provided in the Bottom of this Booklet.
2. The question paper consists of **'50'** objective type questions.
3. Each question has four choices (1), (2), (3) and (4) out of which **ONLY ONE** is correct.
4. Each correct answer carries **4 marks** and each wrong answer **(– 1) Mark**.
5. Use **Black or Blue Ball Point Pen** only for filling particulars.
6. Use of Blank Papers, Clip Boards, Calculator, Log Table, Slide Rule and Mobile or any electronic gadgets in any form is not allowed.
7. In case of any dispute, the answer filled in the OMR sheet available with the institute shall be final.
8. After completion submit the Question Paper back along with the Answer Sheet.

Name: ____________________

Q.1 Suppose that f is a function that is defined for all numbers. Which one of the following conditions assures that f has an inverse function?

(1) The graph of f is symmetric with respect to the y-axis.

(2) The graph of f is concave up.

(3) The function f is stricly increasing function

(4) The function f is continous.

Q.2 The measure of dispersion is

(1) Mean deviation (2) S.D.

(3) Quartile deviation (4) All of these

Q.3 $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a/an

(1) integer only when n is an even integer (2) integer for all $n \in N$

(3) integer only when n is an odd integer (4) not integer

Q.4 The angles of elevation of the top of a TV tower from three points A, B and C in a straight line (in the horizontal plane) through the foot of tower are α, 2α and 3α respectively. If AB = a, the height of tower is

(1) $a \tan \alpha$ (2) $a \sin \alpha$ (3) $a \sin 2\alpha$ (4) $a \sin 3\alpha$

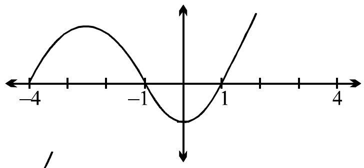
Q.5 Let $|z_1| = |z_2| = \frac{c}{2}$ then $|z_1 + z_2|^2 + |z_1 - z_2|^2 =$

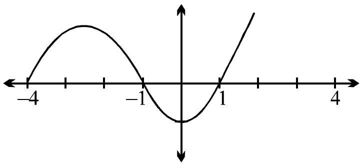
(1) c^2 (2) $c^2/2$ (3) $2c^2$ (4) none

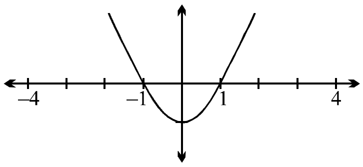
SPACE FOR ROUGH WORK

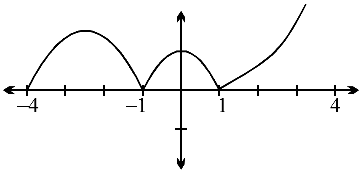
Q.6 The graph of $y = f(x)$ is shown in the figure.

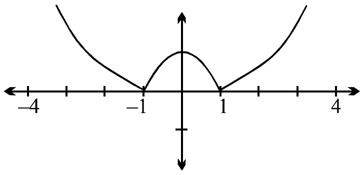
Which of the following could be the graph of $y = f(|x|)$?



(1) 

(2) 

(3) 

(4) 

Q.7 If A is a square matrix of order $n \times n$ and k is a scalar, then adj (kA)

(1) k adj A (2) k^n adj A (3) k^{n-1} adj A (4) k^{n+1} adj A

Q.8 Which of the following is a nilpotent matrix

(1) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ (3) $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Q.9 Seven people are sitting in a theater watching a show. The row they are in contains seven seats. After intermission, they return to the same row but choose seats randomly. What is the probability that neither of the people sitting in the two aisle seats was previously sitting in an aisle seat ?

(1) $\frac{3}{7}$ (2) $\frac{10}{21}$ (3) $\frac{11}{21}$ (4) $\frac{4}{7}$

Q.10 Which one of the complex numbers when added to the complex number $z = 4e^{\frac{5\pi}{4}i}$ will result in a sum having the greatest distance from the origin of the argand plane?

(1) $4e^{\frac{\pi}{4}i}$ (2) $\frac{1+i}{\sqrt{2}}$ (3) $2e^{-\frac{3\pi}{4}i}$ (4) $\frac{5+i}{\sqrt{2}}$

Q.11 If $f(x) = \begin{cases} |x^2 - 5x + 6| & \text{for } x > 2 \\ ax - b & \text{for } x \le 2 \end{cases}$ is continuous at x = 2, then the values of 'a' & 'b' can not be :

(1) a = 1 ; b = 2 (2) a = 0 ; b = 0 (3) a = 2 ; b = 1 (4) a = 5 ; b = 10

SPACE FOR ROUGH WORK

Q.12 The value of $\lim_{x\to\infty} \dfrac{2x-3\sin^{-1}x}{3x+2\tan^{-1}x}$ is equal to

(1) $\dfrac{2}{3}$ (2) 0 (3) 1 (4) 5

Q.13 If $f(x)=\dfrac{\sin(2\pi[\pi^2 x])}{5+[x]^2}$

[**Note:** [k] denotes the largest integer less than or equal to k.]
Then $f(x)$, is
(1) discontinuous at some x.
(2) continuous at all x, but the derivative $f'(x)$ does not exist for some x.
(3) $f'(x)$ exists for all x, but $f''(x)$ does not exist at some x.
(4) $f''(x)$ exist for all x.

Q.14 The value of $\int_{0}^{\pi} \dfrac{x\sin x}{1+\cos^2 x}dx$ is

(1) $\dfrac{\pi^2}{4}$ (2) $\dfrac{\pi^2}{8}$ (3) $\dfrac{\pi^2}{2}$ (4) none

Q.15 The vectors $\vec{a}=(2-x)\hat{i}+2\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}+(2-y)\hat{j}+2\hat{k}$, $\vec{c}=2\hat{i}+2\hat{j}+(2-z)\hat{k}$ and $\vec{d}=\hat{i}+\hat{j}+\hat{k}$ are coplanar, then

(1) $\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=1$ (2) $\dfrac{1}{1-x}+\dfrac{1}{1-y}+\dfrac{1}{1-z}=1$

(3) $x+y+z=1$ (4) $\dfrac{1}{x-2}+\dfrac{1}{y-2}+\dfrac{1}{z-2}=1$

Q.16 If $\vec{a}=x\hat{i}-2\hat{j}+5\hat{k}$ and $\vec{b}=\hat{i}+y\hat{j}-z\hat{k}$ are linearly dependent, then the value of $\dfrac{xy^2}{z}$ equals

(1) $\dfrac{4}{5}$ (2) $\dfrac{-3}{5}$ (3) $\dfrac{3}{5}$ (4) $\dfrac{-4}{5}$

Q.17 The equation of a circle is $x^2+y^2=25$. The equation of its chord whose middle point is $(1,-2)$ is given by
(1) $2x-y-4=0$ (2) $5x+y-3=0$ (3) $x-2y-5=0$ (4) $x+y+1=0$

SPACE FOR ROUGH WORK

Q.18 If algebraic sum of distances of a variables line from points A (3, 0), B (0, 3) and C (– 3, – 3) is zero, then the line passes through the fixed point

(1) (3, 3) (2) (–1, – 1) (3) (1, 1) (4) (0, 0)

Q.19 Number of solutions which simultaneously satisfy the system $\sin^2 x + \sin^2 2x = \sin^2 3x$, $\frac{-\pi}{2} \le x \le \pi$, is

(1) 3 (2) 6 (3) 10 (4) 7

Q.20 $\int_0^{\pi} \sin x \cos 3x \, dx$ has the value equal to

(1) $\frac{3}{4}$ (2) $\frac{3}{8}$ (3) $\frac{3}{2}$ (4) none

Q.21 The value of the expression $\left(\frac{\sin 3\theta}{\sin\theta}\right)^2 - \left(\frac{\cos 3\theta}{\cos\theta}\right)^2$ when $\theta = 7.5°$ is

(1) $4(\sqrt{3}+1)$ (2) $4(\sqrt{3}-1)$ (3) $2(\sqrt{6}-\sqrt{2})$ (4) $2(\sqrt{6}+\sqrt{2})$

Q.22 If
$2x + y - z = 5$
$3x - 2y + 2z = -3$ and
$x - 3y - 3z = -2$

then the value of y satisfying the system of equations is

(1) – 1 (2) 2 (3) 1 (4) 0

Q.23 Let A and B are two events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$. If the probability of atleast one of their occuring is $\frac{2}{3}$ then $P(A^C \cup B^C)^C$ has the value equal to

(1) $\frac{1}{4}$ (2) $\frac{1}{9}$ (3) $\frac{1}{6}$ (4) $\frac{1}{5}$

Q.24 There are exactly two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose distance from the centre of the ellipse are greatest and equal to $\sqrt{\frac{a^2 + 2b^2}{2}}$. Eccentricity of this ellipse is equal to

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{\sqrt{3}}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{\frac{2}{3}}$

SPACE FOR ROUGH WORK

Q.25 The position vectors of the 3 angular points A, B, C of a triangle are (3, 2, – 3) ; (5, 1, – 1) and (1, –2, 1) respectively. If the bisector of the angle A meets the side BC at D then the position vector of the point D are

(1) $\frac{1}{3}\left(11\hat{i}+\hat{k}\right)$ (2) $\frac{1}{3}\left(11\hat{i}-\hat{k}\right)$ (3) $\frac{1}{3}\left(11\hat{i}+\hat{j}-\hat{k}\right)$ (4) none

Q.26 **Statement-1:** The solution of the equation $x \sin\theta\, d\theta + (x^3 - 2x^2 \cos\theta + \cos\theta)dx = 0$ is

$2\cos\theta = x + cx\, e^{-x^2}$.

Statement-2: Integrating factor $= \frac{e^{x^2}}{x}$.

(1) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

Q.27 **Statement-1:** $\int_0^{10\pi} (|\sin x| + |\cos x|)dx = 40$

Statement-2: $|\sin x| + |\cos x|$ is a periodic function of period $\frac{\pi}{2}$.

(1) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

Q.28 **Statement-1:** If $2f(x) + 3f\left(\frac{1}{x}\right) = \frac{1}{x} - 2, x \neq 0$, then $\int_1^2 f(x)\,dx = -\frac{2}{5}\log 2 + \frac{1}{2}$.

Statement-2: $f(x) = -\frac{2}{5x} + \frac{3x}{5} - \frac{2}{5}$.

(1) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

SPACE FOR ROUGH WORK

Q.29 **Statement-1:** If $f(x) = \dfrac{x+1}{x-1}$, $x \neq 1$, then $(fofofofof)(x) = f(x)$.

Statement-2: $fof(x) = f(x)$.

(1) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

Q.30 **Statement-1:** The value of $\lim\limits_{x\to\infty} \dfrac{3^{x+1}-5^{x+1}}{3^x-5^x}$ is 5.

Statement-2: $\lim\limits_{n\to\infty} a^n = 0$ if $0 < a < 1$.

(1) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

Q.31 If $x + 7y = 29$ is an equation of the line normal to the graph of f at the point (1, 4), then $f'(1)$ is equal to

(1) $\dfrac{1}{7}$ (2) $\dfrac{-1}{7}$ (3) -7 (4) 7

Q.32 The equation $|z-4| = |z-2|$ represents :
(1) real part (z) > 0 (2) real part (z) < 0 (3) straight line (4) none

Q.33 If $\int f(x)\sin x\,dx = -f(x)\cos x + \int 3x^2 \cos x\,dx$, then f(x) could be
(1) $3x^2$ (2) x^3 (3) $-x^3$ (4) $-3x^2$

Q.34 The function $f : [2, \infty) \to Y$ defined by $f(x) = x^2 - 4x + 5$ is both one-one and onto if :
(1) Y = R (2) $Y = [1, \infty)$ (3) $Y = [4, \infty)$ (4) $Y = [5, \infty)$

Q.35 The functions, $f(x) = \cos^{-1}(4x^3 - 3x)$ and $g(x) = 3\cos^{-1} x$ are identical in the interval

(1) $[-1, 1]$ (2) $\left[-1, \dfrac{-1}{2}\right]$ (3) $\left[\dfrac{-1}{2}, \dfrac{1}{2}\right]$ (4) $\left[\dfrac{1}{2}, 1\right]$

SPACE FOR ROUGH WORK

Q.36 Number of integral values of 'a' for which $\int_1^2 (a^2 + 4(1-a)x + 4x^3)\, dx < 13$, is

(1) 0 (2) 1 (3) 3 (4) infinite

Q.37 Let A and B are two events defined on a sample space such that

$P(A) = \frac{3}{8}$; $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, then

(1) P(B/A) > P(A/B) (2) P(A/B) > P(B/A)
(3) P(A/B) = P(B/A) (4) A and B are independent

Q.38 The largest integral value of k for which the graph of the function $f(x) = x^2 - 2(k+2)x + 12 + k^2$ lies completely above or below the x-axis is

(1) 1 (2) – 2 (3) 2 (4) 0

Q.39 The area enclosed by the curve $xy = 3$ and the line $y = 4 - x$, is

(1) $4 - ln3$ (2) $6 - 3\, ln3$ (3) $3\, ln3$ (4) $4 - 3\, ln3$

Q.40 The value of 'm' for which the angle between the vectors $\vec{x} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{y} = m\hat{i} + \hat{j} - \hat{k}$ is equal to $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$ is

(1) $\frac{1}{2}$ (2) $\pm\sqrt{\frac{2}{3}}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\frac{2}{3}$

Q.41 Distance of the point (1, – 2, 3) from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is

(1) $\frac{1}{7}$ (2) 2 (3) $\frac{4}{7}$ (4) 1

Q.42 Range of the function, $y = \frac{x^2 - x + 1}{2x^2 - 2x + 3}$ is

(1) $\left[\frac{3}{10}, \frac{1}{2}\right)$ (2) $\left[\frac{1}{3}, \frac{1}{2}\right)$ (3) (2, 3] (4) $\left[\frac{1}{2}, 2\right)$

Q.43 Suppose $F(x) = f(g(x))$ and $g(3) = 5, g'(3) = 3, f'(3) = 1, f'(5) = 4$. Then the value of $F'(3)$, is

(1) 15 (2) 12 (3) 9 (4) 7

SPACE FOR ROUGH WORK

Q.44 An aeroplane flying at a height of 300 m above the ground passes vertically above another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Then the height of the lower plane from the ground is

(1) $100\sqrt{3}$ m (2) $\frac{100}{\sqrt{3}}$ m (3) 50 m (4) $150(\sqrt{3}+1)$ m

Q.45 The foot of perpendicular from (1, 2, 6) to line $\frac{x-3}{2} = \frac{y+1}{2} = \frac{z-1}{-1}$ is the point (α, β, γ). Then $(\alpha + \beta + \gamma)$ is equal to

(1) – 3 (2) 2 (3) 7 (4) 6

Q.46 It is given that Rolle's theorem holds for function $f(x) = x^3 + bx^2 + cx$, for $1 \le x \le 2$ at the point $x = 4/3$, then $(b + c)$ is equal to

(1) 3 (2) 2 (3) 1 (4) 0

Q.47 If $\lim\limits_{x\to 0}\frac{\tan x - \sin x}{x^n}$ $(n \in N)$ exists then the sum of all possible values of n is

(1) 4 (2) 5 (3) 6 (4) 10

Q.48 If the equations $ax^2 + 2bx + c = 0$ and $ax^2 + 2cx + b = 0$, $b \ne c$, have a common root, then $\frac{a}{b+c} =$

(1) 1 (2) – 4 (3) $-\frac{1}{2}$ (4) $-\frac{1}{4}$

Q.49 Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ 3 & 7 & -2 \end{bmatrix}$. The second column of A^{-1} is

(1) $\begin{bmatrix} -2 \\ 0 \\ -3 \end{bmatrix}$ (2) $\begin{bmatrix} 2/5 \\ 0 \\ 3/5 \end{bmatrix}$ (3) $\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$ (4) A is not invertible

Q.50 If ω be a complex cube root of unity, then the value of $\frac{1}{1+2\omega} - \frac{1}{1+\omega} + \frac{1}{2+\omega}$ is

(1) 0 (2) 1 (3) – 1 (4) none

SPACE FOR ROUGH WORK